EFFECTS OF COMPRESSIBILITY AND PRESSURE GRADIENT ON THE CRITICAL-ROUGHNESS REYNOLDS NUMBER

E. U. Repik

UDC 532.011.18

Results are shown of an experimental study concerning the effects of compressibility and of a longitudinal pressure gradient on the critical-roughness Reynolds number.

Since in engineering one never deals with perfectly smooth surfaces, hence for practical calculations of a turbulent boundary layer it is very important to know the critical-roughness Reynolds number and thus to establish the maximum height K_c of asperities (grains) at which the skin friction will still be the same as in the case of a smooth surface: $C_{Fr}/C_{Fsm} = 1$. A surface with $K < K_c$ is conventionally considered aerodynamically smooth.

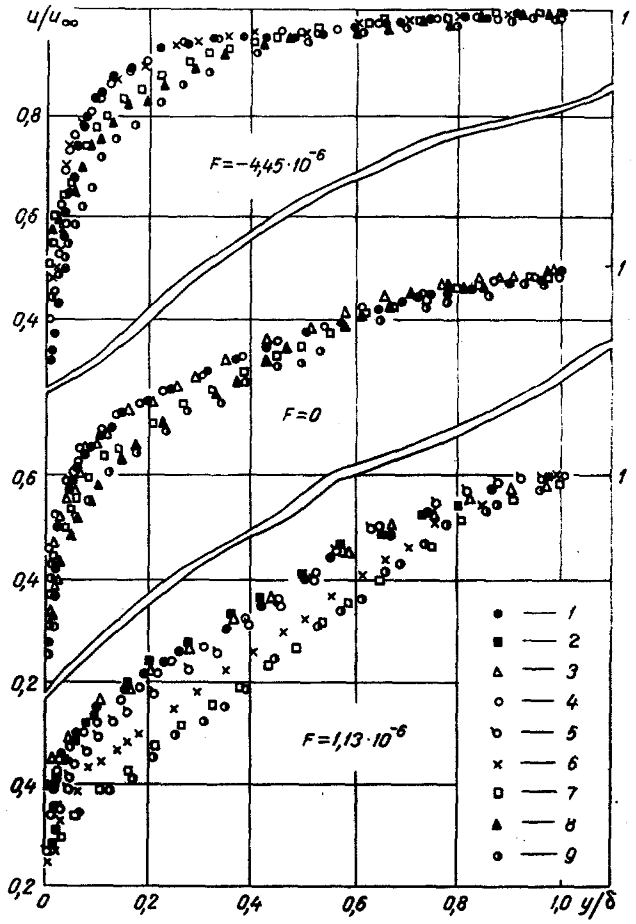

Fig. 1. Velocity profile of the boundary layer at a rough surface ($u_\infty = 11$ m/sec): K = 0 (1), 0.04 mm (2), 0.10 mm (3), 0.16 mm (4), 0.25 mm (5), 0.32 mm (6), 0.40 mm (7), 0.50 mm (8), 0.80 mm (9).

Many studies have dealt with the problem of determining Re_c, but in all cases the main concern was the flow of fluids through rough pipes [1]. Little experimental work has been done concerning the flow in a boundary layer at a rough surface. For instance, the effects of compressibility and of a longitudinal pressure gradient on the critical-roughness Reynolds number have remained almost entirely unexplored. As a consequence, in semi-empirical theories of the turbulent boundary layer at a rough surface (e.g., in [2]) the viewpoint still prevails that the effect of compressibility on the value of Re_c can be eliminated by referring the kinematic viscosity in the Reynolds number to conditions at the surface. In this case, then, the value of Re_c is the same for a compressible and an incompressible fluid.

The numerical values of Re_c based on various studies differ appreciably. Thus, it was suggested in [3] that $Re_c = 10$ independently of the Mach number, while $Re_c = 6$ was assumed in [2]. It is to be noted that these values are based on tests at supersonic values of the Mach number. No experiments with an incompressible fluid are described in either of these references. That Re_c should be independent of the Mach number when the kinematic viscosity is referred to conditions at the surface, as has been assumed, does not at all obviously follow

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 3, pp. 427-433, March, 1973. Original article submitted April 11, 1972.

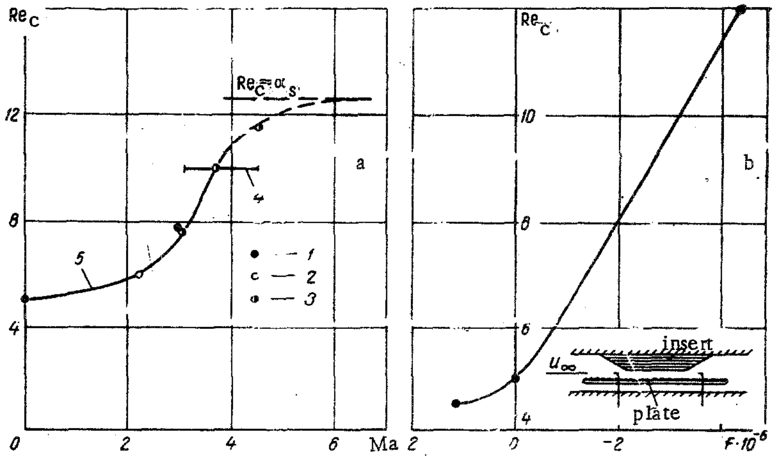

Fig. 2. Critical-roughness Reynolds number as a function of the Mach number (a) and as a function of the longitudinal pressure gradient (parameter F) (b): according to the author's test data (1), according to [2] (2), according to [3] and evaluated by the author (Fig. 27 in [3]) (3), averaged by F. E. Goddard [3] (4), according to formula (2) (5).

from the semiempirical theories of the turbulent boundary layer, inasmuch as referring the physical properties of a gas to conditions at the surface in the extreme case of a smooth surface does not, as is well known, eliminate but only slightly weakens the effect of compressibility on the characteristics of that boundary layer.

In experimental studies concerning the effect of roughness on the characteristics of a boundary layer there arise difficulties in treating the geometry of roughness elements and their distribution across the immersed surface, inasmuch as roughness can appear technically in many modes. This has made it necessary to introduce the concept of equivalent roughness on the basis of sand roughness as the standard.

In our tests roughness was produced by pasting on the immersed surface an abrasive cloth on a paper base (crocus) with fourteen different granular structure grades. The resulting roughness was one of maximum density and identifiable by the mean height of asperities (grains) alone [1].

According to measurements of the roughness profile on the abrasive cloth with a dial indicator, the mean squared heights of asperities (grains) in our tests were 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.16, 0.20, 0.25, 0.32, 0.40, 0.50, and 0.80 mm.

The mean coefficient of "rough" friction was in our tests determined from the velocity profile measurements in the boundary layer, on the basis of the relation $C_F = 2\delta^{**}/x$, with the y-coordinate measured from the top level of the asperities. In this way the velocity profiles near a rough and near a smooth surface could be compared best [4].

A typical velocity profile of the boundary layer is shown in Fig. 1 for various values of the longitudinal pressure gradient. The velocity profile at a rough surface deviates from the velocity profile at a smooth surface ($K = 0$) only when $K > K_c$. According to the graphs, this critical roughness is 0.32 mm at a negative pressure gradient ($F = -4.45 \cdot 10^{-6}$) and approximately half as high 0.16 mm at a positive pressure gradient ($F = 1.13 \cdot 10^{-6}$).

In order to establish the critical-roughness Reynolds number Re_c, it is necessary to know the coefficient of local "rough" friction at an aerodynamically smooth surface ($K \leq K_c$), which has been determined at $dP/dx = 0$ both directly as a friction force with the aid of a "floating" probe and sensitive electromagnetic scales and indirectly with total-head Pitot tubes installed at the surface [5]. In the presence of longitudinal pressure gradients produced by means of specially shaped inserts,* however, in the active channel of a 110×100 mm pipe (Fig. 2b) the local friction at the aerodynamically smooth surface was measured with a thermal friction probe [7] as well as on the basis of the experimentally established velocity profile of the boundary layer and the Ludwieg–Tillman formula [8]:

*The inserts were contoured following the recommendations in [6].

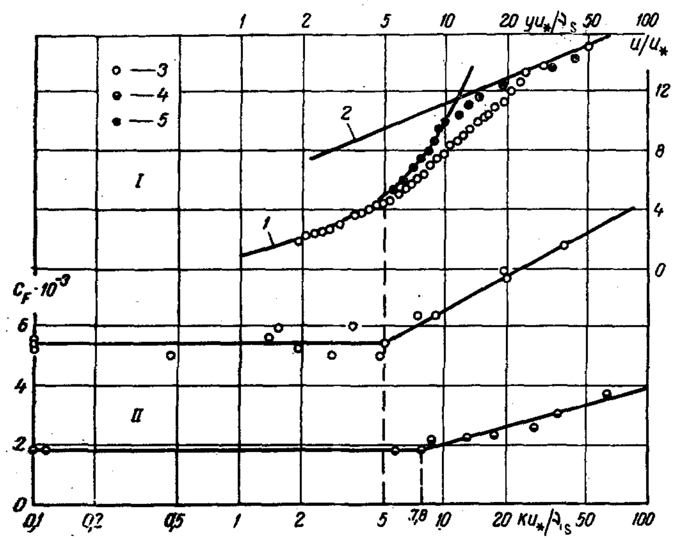

Fig. 3. Velocity profile of the laminar sublayer of a boundary layer at a smooth surface (I) and mean coefficient of skin friction at a rough surface, as a function of the asperity height (II): $u/u_* = yu_*/\nu_s$ (1), $u/u_* = 5.5 + 5.75 \log yu_*/\nu_s$ (2), incompressible fluid with $Re_x = 360{,}000$ (3), Ma = 3 and $Re_x = 7{,}160{,}000$ (4), Ma = 6 and Re** = 2800 at $\overline{T}_s = 0.45$ (5).

$$C_f = 0.246 \cdot 10^{-0.678H} \, \mathrm{Re}^{**\,-0.268}. \tag{1}$$

In order to answer the question as to whether the compressibility of a gas affects the critical-roughness Reynolds number, the author has analyzed the boundary layer at a flat plate with the same roughness characteristics in a stream of incompressible fluid as well as in a supersonic stream with the Mach number Ma = 3. The results are shown in Fig. 2a. According to these test data, the critical-roughness Reynolds number $Re_c = 5$ for a boundary layer of incompressible fluid and $Re_c = 7.8$ when Ma = 3, thus obviously increases as the Mach number becomes higher – even with the kinematic viscosity in the expression for the Reynolds number referred to conditions at the surface [2, 3]. If we dismiss the hypothesis that referring the kinematic viscosity to conditions at the surface will eliminate the effect of compressibility, and if we transpose on Fig. 2 the test data from [2] and [3] for the same values of the Mach number, then all test points will fit on a universal curve of Re_c as a function of the Mach number approximately described by the equation

$$\mathrm{Re}_c = 0.175M^3 - 0.5M^2 + 0.7M + 5 \quad \text{for } 0 < M < 4. \tag{2}$$

With Re_c assumed independent of the Mach number, therefore, one obtains a lower maximum allowable height of asperities. It is also noteworthy that, with Re_c dependent on the Mach number, the roughness function

$$f(\mathrm{Re}_c) = A \log \left(\frac{u_* K}{\nu_s}\right)_{K > K_c} + B$$

ceases to be universal – as is usually stipulated in semiempirical theories as a basis for plotting the logarithmic velocity profile of a boundary at a rough surface [2].

The existence of a positive pressure gradient ($F = 1.13 \cdot 10^{-6}$, $\Delta = 0.0216$) in our tests caused an insignificant reduction of the critical-roughness Reynolds number ($Re_c \approx 4.5$) below its value for a zero-gradient flow, while a negative longitudinal pressure gradient ($F = -4.45 \cdot 10^{-6}$, $\Delta = -0.0303$) in our tests caused the critical-roughness Reynolds number to increase appreciably up to $Re_c = 12$ (Fig. 2b).

The higher value of Re_c at a negative pressure gradient explains why under this condition the drag of bodies is often found in practice to increase slower with a deteriorating surface finish than would follow according to the prevalent hypothesis that Re_c is independent of the pressure gradient.

It is well known that the extent to which roughness affects skin friction depends on the ratio of the asperity height to the thickness of the laminar sublayer of the boundary layer at a smooth surface [1] and, therefore, the critical-roughness Reynolds number is

$$\frac{K_c u_*}{\nu_s} = \alpha_s \frac{K_c}{\delta_l}. \tag{4}$$

Experiments performed by J. Nikuradse [9] have shown that the effect of roughness on skin friction does not become apparent as long as all asperities (grains) remain contained within the laminar sublayer. There are no direct measurement data available on how deeply below the outer edge of the laminar sublayer the asperities (grains) must remain. Our measurements here of both the thickness of the laminar sublayer at a smooth surface ($K = 0$) and of the critical asperity height on a rough surface have shown that the ratio K_c/δ_l is not a constant quantity but a function of the Mach number and of the pressure gradient. Thus, $K_c/\delta_l = 0.51$ in a zero-gradient stream of an incompressible fluid and $K_c/\delta_l = 0.75$ at $Ma = 3$, i.e., K_c increases faster than the thickness of the laminar sublayer with an increasing Mach number. On the basis of relation (4), for a thermally insulated plate with the dimensionless thickness parameter of the laminar sublayer α_s only weakly dependent on the Mach number Ma, the effect of compressibility on the value of Re_c is determined by the change in the ratio K_c/δ_l. In the case of a flow with longitudinal pressure gradients, on the other hand, Re_c is basically determined by the change in α_s [10] – although at a negative pressure gradient the ratio K_c/δ_l increases too. We thus have $K_c/\delta_l = 0.62$ for $F = -4.45 \cdot 10^{-6}$, while $K_c/\delta_l = 0.52$ for $F = 1.13 \cdot 10^{-6}$ as in the case of $dP/dx = 0$.

As has been mentioned already, the height K_c is commensurable with the dimension of that zone of the laminar sublayer* at a smooth surface where the velocity profile is linear and approaches the velocity profile of a laminar boundary layer. According to Fig. 3, for instance, in an incompressible fluid with $dP/dx = 0$ the velocity profile at a smooth surface remains linear up to $yu_*/\nu_s = 5$ but is $K_c u_*/\nu_s$ at a rough surface. As the Mach number becomes higher, the velocity profile becomes linear over an increasing zone of the laminar sublayer until it extends already up to $yu_*/\nu_s = 10$ at $Ma = 6$. In this case the ratio K_c/δ_l should approach unity and, consequently, the critical-roughness Reynolds number will be determined by the dimensionless thickness parameter of the laminar sublayer α_s, i.e., $Re_c \approx \alpha_s$ (Fig. 2a).

In view of this, the following observations may be of interest. It is well known that roughness has almost no effect on skin friction in a laminar boundary layer, but instead facilitates a sooner transition from laminar to turbulent flow. One would expect that, by analogy, skin friction will not depend on the roughness in a turbulent boundary layer as well, if the asperities (grains) remain contained within the zone of nearly laminar flow in the boundary layer. If the asperities (grains) protrude beyond this zone of a linear velocity profile, however, then the laminar sublayer becomes disrupted and the skin friction increases.

The trends which have been described here allow one to estimate how much the critical-roughness Reynolds number depends on the flow conditions in a liquid or a gas stream, not only on the basis of skin-friction measurements at a rough surface but also from the results of a flow analysis in the laminar sublayer at a perfectly smooth surface.

NOTATION

C_F	is the mean coefficient of skin friction;
C_f	is the local coefficient of skin friction;
τ	is the shear stress;
$Re_x = ux/\nu_\infty$	is the Reynolds number referred to the distance from the frontal plate edge;
$Re^{**} = u\delta^{**}/\nu_\infty$	is the Reynolds number referred to the momentum thickness;
$Re_c = u_* K_c/\nu_s$	is the critical-roughness Reynolds number;
$F = (\nu_\infty/\rho_\infty)[(dP/dx)/u_\infty^3]$,	
$\Delta = [(\nu_\infty dP/dx)/(\rho_\infty u_*^3)]$	are the parameters of the longitudinal pressure gradient;
Ma	is the Mach number;
$u_* = \sqrt{\tau_s/\rho}$	is the dynamic velocity;
u	is the velocity;
u/u_*	is the dimensionless velocity;
dP/dx	is the pressure gradient;

*We consider here the two-layer model of a boundary layer.

x	is the distance along the surface from the front edge;
y	is the distance from the immersed surface along a normal to it;
δ	is the thickness of a boundary layer;
δ_l	is the thickness of the laminar sublayer of a boundary layer;
$\delta^* = \int_0^\delta (1-\rho u/\rho_\infty u_\infty)dy$	is the displacement thickness;
$\delta^{**} = \int_0^\delta \rho u/\rho_\infty u_\infty (1-u/u_\infty)dy$	is the momentum thickness;
yu_*/ν_s	is the dimensionless distance from the surface;
$\alpha_s = u_*\delta_l/\nu_s$	is the thickness of the laminar sublayer of a boundary layer;
$H = \delta^*/\delta^{**}$	is the form factor of the velocity profile;
K	is the mean squared height of asperities;
K_c	is the critical mean squared height of asperities;
T	is the temperature;
$\bar{T}_s = T_s/T_0$	is the temperature factor;
ν	is the kinematic viscosity of the fluid;
ρ	is the density of the fluid.

Subscripts

s	refers to conditions at the surface;
∞	refers to conditions at the outer edge of boundary layer;
r	refers to rough surface;
sm	refers to smooth surface.

LITERATURE CITED

1. G. Schlichting, Theory of the Boundary Layer [Russian translation], Izd. Nauka, Moscow (1969).
2. F. W. Fenter, Preprint No. 837, Inst. Aeronaut. Sci., New York (1958).
3. F. E. Goddard, J. Aero-Space Sci., 26, No. 1, 1-15 (1959).
4. Lin Tsa Tsao (editor), Turbulent Flow and Heat Transfer [Russian translation] Izd. Inostr. Lit., Moscow (1963).
5. E. U. Repik and V. N. Tarasova, Trudy TsAGI, No. 1218, Moscow (1970).
6. J. F. Nash, A. R. C. Current Papers, No. 835 (1966).
7. D. A. Spence and G. L. Brown, J. Fluid Mechan., 33, Part 4 (1968).
8. H. Ludwieg and W. Tillman, NASA Tech. Mem. 1285 (1950).
9. J. Nikuradse, Forschungsarbeiten Ingenieurenwesen [German], 356 (1932).
10. E. U. Repik, Trudy TsAGI, No. 1218, Moscow (1970).